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ABSTRACT

Canonical correlation analysis is a powerful statistical method subsuming other parametric significance tests as special cases, and which can often best honor the complex reality to which most researchers wish to generalize. However, it has been suggested that the canonical correlation coefficient is positively biased. A Monte Carlo study involving 1,000 random samples from each of 64 different population matrices was conducted to investigate bias in both canonical correlation and redundancy coefficients, and to provide an empirical basis for isolating an appropriate correction formula. Results indicate that the Wherry correction, first suggested for use with the multiple correlation coefficient, provides a reasonable correction that is sensitive to those factors most affecting bias. The results also indicate that canonical results are not as positively biased as some researchers have believed, especially if sample size is at least 10 subjects per variable or effect sizes are moderate or large. (Thirteen tables and three appendices present study data, and a 39-item list of references is provided.) (Author)

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Finding a Correction for the Sampling Error in Multivariate Measures of Relationship: A Monte Carlo Study

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ABSTRACT

Canonical correlation analysis is a powerful statistical method subsuming other parametric significance tests as special cases, and which can often best honor the complex reality to which most researchers wish to generalize. However, it has been suggested that the canonical correlation coefficient is positively biased. A Monte Carlo study involving 1,000 random samples from each of 64 different population matrices was conducted to investigate bias in both canonical correlation and redundancy coefficients, and to provide an empirical basis for isolating an appropriate correction formula. Results indicate that the Wherry correction, first suggested for use with the multiple correlation coefficient, provides a reasonable correction that is sensitive to those factors most affecting bias. The results also indicate that canonical results are not as positively biased as some researchers have believed, especially if sample size is at least 10 subjects per variable or effect sizes are moderate or large.

Multivariate statistics have been available to researchers for many years, although even today "there are many articles in the research literature in which multiple univariate statistics are calculated rather than a single multivariate analysis; for instance, one article may report 50 _t_-tests rather than one MANOVA" (Moore, 1983, p. 307). McMillan and Schumacher (1984) isolated one reason why some researchers have hesitated to use multivariate statistical methods:

> The statistical procedures for analyzing many variables at the same time have been available for many years, but it has only been since the computer age that researchers have been able to utilize these procedures. There is thus lag in training of researchers that has militated against the use of these more sophisticated procedures. There are in evidence more each year in journals, however... (p. 270)

Hinkle, Wiersma and Jurs (1979) concurred, noting that "it is becoming increasingly important for behavioral scientists to understand multivariate procedures even if they do not use them in their own research." And recent empirical studies of research practice do confirm that multivariate methods are employed with some regularity in behavioral research (Elmore & Woehlke, 1988; Gaither & Glorfeld, 1985; Goodwin & Goodwin, 1985).

There are two reasons why multivariate methods are so important in behavioral research, as noted by Thompson (1986c) and by Fish (1988). First, _multivariate methods control the inflation of Type I "experimentwise" error rates_. Most researchers are familiar with "testwise" alpha. But while "testwise" alpha refers to the probability of making a Type I error for a _given hypothesis test_, "experimentwise" error rate refers to the probability of having made a Type I error _anywhere within the study_. When only one hypothesis is tested for a given group of people in a study, "experimentwise" error rate will exactly equal the "testwise" error rate.

But when more than one hypothesis is tested in a given study, the two error rates will not be equal. Witte (1985, p. 236) explains the two error rates using an intuitively appealing example involving a coin toss. If the toss of heads is equated with a Type I error, and if a coin is tossed only once, then the probability of a head on the one toss and of at least one head within the set of one toss will both equal 50%. But if the coin is tossed three times, even though the "testwise" probability of a head on each given toss in 50%, the "experimentwise" probability that there will be at least one head in the whole set of three flips will be inflated to more than 50%. Researchers control "testwise" error rate by picking small values, usually 0.05, for the "teswtise" alpha. "Experimentwise" error rate can be controlled at the "testwise" level by employing multivariate statistics.

When researchers test several hypotheses in a given study, but do not use multivariate statistics, the "experimentwise" error rate will range somewhere between the "testwise" error rate and the ceiling calculated in the manner illustrated in Table 1. Where the experimentwise error rate will actually lie will depend upon the degree of correlation among the dependent variables in the study. Because the exact rate in a practical sense is readily estimated only when the dependent variables are perfectly correlated (and "experimentwise" error will equal the "testwise" error) or are perfectly uncorrelated (and "experimentwise" error will equal the ceiling calculated in the manner illustrated in Table 1), it is particularly disturbing that the researcher may not even be able to determine the exact "experimentwise" error rate in some studies!

INSERT TABLE 1 ABOUT HERE.

Paradoxically, although the use of several univariate tests in a single study can lead to too many hypotheses being spuriously rejected, as reflected in inflation of "experimentwise" error rate, it is also possible that the failure to employ multivariate methods can lead to a failure to identify statistically significant results which actually exist. Fish (1988) provides a data set illustrating this equally disturbing possibility. The basis for this paradox is beyond the scope of the present treatment, but involves the second major reason why multivariate statistics are so important.

Multivariate methods are often vital in behavioral research because <u>multivariate methods best honor the reality to which the researcher is purportedly trying to generalize</u>. Since significance testing and error rates may not be the most important aspect of research practice (Thompson, 1988b), this second reason for employing multivariate statistics is actually the more important of the two grounds for using these methods.

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Thompson (1986c, p. 9) notes that the reality about which most researchers wish to generalize is usually one "in which the researcher cares about multiple outcomes, in which most outcomes have multiple causes, and in which most causes have multiple effects." As Hopkins (1980, p. 374) has emphasized:

> These multivariate methods allow understanding of relationships among several variables not possible with univariate analysis... Factor analysis, canonical correlation, and discriminant analysis--and modifications of each procedure--allow researchers to study complex data, particularly situations with many interrelated variables. Such is the case with questions based in the education of human beings.

Similarly, McMillan and Schumacher (1984) argue that:

> Social scientists have realized for many years that human behavior can be understood only by examining many variables at the same time, not by dealing with one variable in one study, another variable in a second study, and so forth... These [univariate] procedures haved failed to reflect our current emphasis on the multiplicity of factors in human behavior... In the reality of complex social situations the researcher needs to examine many variables simultaneously. (pp. 269-270)

One of the most useful multivariate methods is canonical correlation analysis, a statistical procedure first conceptualized by Hotelling (1935). However, some researchers have suggested that canonical correlation coefficients may be seriously positively biased estimates of true population values. This argument has been offered based on both theoretical grounds (e.g., Cooley & Lohnes, 1976, p. 212) and some limited previous empirical research (cf. Sweet, 1973). But, as Cliff (1987, p. 446) notes, the degree of capitalization on sampling error in the calculation of *Rc* is still not understood.

The present study was conducted for two purposes. First, the study was conducted to explore the degree of bias in two commonly used measures of relationship derived through canonical correlation analysis. Second, the study was conducted to provide a correction factor to apply to canonical results so that more accurate findings can be used as the basis for interpretation. Such correction formulae are fairly commonly applied with some research results, as in the Olkin-Pratt correction for bias in the multiple correlation coefficient (Olkin & Pratt, 1958). In the present study the correction factor was empirically derived. Tatsuoka (1973) provides an example of empirical efforts to derive correction factors for multivariate results.

The Basic Logic of Canonical Analysis

The Canonical Correlation Coefficient (Rc)

Although canonical analysis is explained in several recent texts (Marascuilo & Levin, 1983; Thompson, 1984), some readers may appreciate a brief discussion of the logic of canonical analysis, prior to the presentation of the results of computer simulations reported here. Table 2 presents the simplest case of a true multivariate correlation analysis, since there are two variables in both the predictor ("A" and "B") and the criterion ("X" and "Y") variable sets. The table also presents the $\underline{Z}$-score equivalents of the raw scores of the 12 hypothetical subjects on all four variables. The full canonical results associated with the Table 2 data are presented in Table 3.

INSERT TABLES 2 AND 3 ABOUT HERE.

The function coefficients presented in Table 3 are equivalent to multiple regression beta weights, factor analysis pattern coefficients ("loadings"), and discriminant analysis function coefficients. Like all these weights, function coefficients are the best possible weights for a given data set for a given purpose. In the case of canonical function coefficients, no other weights can be derived for a given data set to yield a larger correlation between variable sets. Thompson and Borrello (1985) provide more detail on the equivalence of coefficients across methods, an equivalence that is to some degree masked by the unfortunate but traditional use of different names to refer to statistical entities that are in fact the same across analytic methods.

The function coefficient weights in Table 3 can be applied to the "observed" $\underline{Z}$-scores reported in Table 2 to create "latent" or "synthetic" variables scores for each of the 12 subjects on each of the two canonical functions reported in Table 3. For example, the function coefficients for criterion variables "X" and "Y" on Function I were, respectively, 0.511 and 0.867. The first subject's $\underline{Z}$-scores for "ZX" and "ZY" were -1.525 and 1.248, respectively. Thus, the application of these weights to these observed *Z*-scores ((0.511 x -1.525) + (0.867 x 1.248)) yields a "latent" canonical composite score ("C1") of 0.303 for this subject, as reported in Table 2. Scores for the canonical composite for the predictor variable set ("P1") are computed in an analogous manner.

The Pearson product-moment correlation between latent criterion and predictor variables "C1" and "P1" is the canonical correlation coefficient (*Rc*) associated with the first canonical function reported in Table 3. Similarly, the bivariate correlation between "C2" and "P2" is *Rc* for the second canonical function.

Correlating observed with latent variables yields coefficients that can be very useful in interpreting canonical results. For example, correlating "X" (or "ZX") with the canonical composite scores associated with the variable's own variable set ("C1") yields what is called a structure coefficient for the variable "X" on canonical Function I. Structure coefficients inform the researcher regarding the nature of the latent canonical variable (e.g., "C1"), and are often vital in interpreting canonical results (Thompson, 1987).

Similarly, correlating the observed variable "X" (or "ZX") with the latent variable for the other variable set on Function I ("P1") helps inform the researcher about the meaning of the latent variable associated with scores for the canonical composite for the predictor variables on Function I. Correlation coefficients between observed and latent variables computed across variable sets are called index coefficients (Thompson, 1984, pp. 30-31), and are also very important in interpretation (Timm & Carlson, 1976, p. 161). Table 4 indicates which canonical coefficients are computed by correlating various combinations of observed and latent variables.

INSERT TABLE 4 ABOUT HERE.

<u>The Canonical Redundancy Coefficient (Rd)</u>

If the squared structure coefficients for a given variable set on a given function are summed and the averaged, the result is called an adequacy coefficient (Thompson, 1984, pp. 24-25). Adequacy coefficients inform the researcher how "adequately", on the average, a given set of latent canonical composite scores do with respect to representing all the variance in the observed variables in the same variable set. Stewart and Love (1968) suggested that multiplying the adequacy coefficient for a given variable set times the squared <u>Rc</u> for a given function yields what they termed a redundancy coefficient (<u>Rd</u>).

Several researchers have argued that redundancy coefficients are extremely useful in the interpretation of canonical results, and have even argued that redundancy coefficients should be less positively biased than canonical correlation coefficients (Cooley & Lohnes, 1976, p. 212). However, it has been noted that <u>Rc</u>, <u>not Rd</u>, is optimized when canonical function coefficients are computed (Thompson, 1984, p. 27), and that redundancy coefficients are averaged univariate statistics rather than true multivariate statistics (Cramer & Nicewander, 1979, p. 43; Thompson, 1987). These considerations suggest that redundancy coefficients are not usually very useful.

However, Rd is useful in assessing whether canonical results consist of a "g"-function, i.e., a function with a very large Rc and on which all or most variables have very large structure coefficients. Such results are not typically expected in a canonical analysis, although Sexton, McLean, Boyd, Thompson and McCormick (1988) do report canonical results in which a "g"-function was isolated. Therefore, since Rd may at least ocassionally be useful in interpeting canonical results, the sampling bias in both Rc and Rd was investigated in the present study.

Method

Monte Carlo or computer simulation methods have been used for various purposes, including confirming that Yates' correction for contingency table chi-square results is inappropriate (Thompson, 1988a), creating a test statistic for evaluating the statistical significance of correlations of factors across different data sets (Thompson, 1986b), and establishing the magnitudes of distortions introduced when ANOVA is inappropriately used (Thompson, 1986a). One type of Monte Carlo study begins with the creation of a large population of data with known characteristics predetermined by the researcher. Then samples of data are randomly selected and sample results are calculated over and over again, usually 1,000 times for each unique population of data. These results are averaged to determine the degree to which sampling error causes bias in estimates of population parameters.

In the present study it was necessary to create populations of data that were multivariate normally distributed, since statistical significance testing of *Rc* requires this assumption (Thompson, 1984, pp. 16-18). Many researchers are interested in significance tests (Thompson, 1988b), try to meet the multivariate normality assumption in their research, and so this assumption was met here in creating the Monte Carlo populations in order to generalize to contemporary research practice.

The present study was conducted to investigate the impacts of sampling error on *Rc* and *Rd* across variations in *v*ariable and sample (*n*) sizes, and across variations in average within-set correlation sizes and in across-set population correlation sizes. The computer program developed by Morris (1975) was employed to generate populations of *N*=6,000-by-*v* with desired parameters. The specific variations explored in the Monte Carlo study were: (a) use of variable sets consisting of 6+6(*v*=12), 4+4(*v*=8), 4+2(*v*=6), or 10+2(*v*=12) variables; (b) sample sizes (*n*) consisting of 3, 10, 25, or 40 persons per variable; and (c) populations in which all correlations were zero, populations in which within-set correlations were all zero but in which between-set correlations were heterogeneous, populations in which within-set correlations were heterogeneous but in which between-set correlations were all zero, and populations in which all bivariate correlation coefficients were non-zero and homogeneous.

One population was created for each of the four types of correlation matrices. Different subsets of variables were employed to represent variations in variable set sizes. For example, for a given population of data, variable set "A" consisted of variables "1" through "6" being correlated with variables "7" through "12". Variable set "B" consisted of variables "1" through "4" being correlated with variables "9" through "12". Tables 5 and 6 present the population correlation coefficients for each of the four types of correlation matrices based on data for N=6,000 subjects, and the footnotes to the tables further explain which subsets of variables were used to study the effects of the four variations in variable set sizes.

INSERT TABLES 5 AND 6 ABOUT HERE.

Monte Carlo Results

This study considered four types of population matrices, four variations in variable set sizes (12=6+6, 8=4+4, 6=4+2, and 12=10+2), and four variations in number of subjects per variable (3, 10, 25, and 40). Thus, 4x4x4, or 64, sets of analyses were conducted. Since 1,000 random samples were drawn for each of the 64 research situations, each involving a different canonical analysis of sample data, a total of 64,000 canonical correlation analyses provided the basis for the interpretations in the study.

Tables 7 through 10 present the results of these analyses. Matrix identification codes in the tables indicate which variable combinations were involved in a given analysis; for example, results associated with matrix "1A" involved matrix #1 (Table 5) and variables "1" through "6" being related with variables "7" through "12". "NCon" indicates the number of subjects per variable involved in a given analysis. Known population parameters involving all 6,000 subjects are reported to exactly two decimal places. Values reported to three decimals and not in parentheses report the mean deviation from true and known population parameters for a given research situation across 1,000 random samples. Values in parentheses are the standard deviations about mean deviations across 1,000 samples, and are akin to standard errors. For example, as reported in Table 7, for matrix #1, variable combination "A", the known population squared Rc was 0.00. The mean deviation from this true value across 1,000 random samples of size n=36 ("NCon" x number of variables = 3 x (6 + 6)) was an overestimate, -0.447; the standard deviation for this estimate across 1,000 samples in this research situation was 0.091.

INSERT TABLES 7 THROUGH 10 ABOUT HERE.

Tables 7 through 10 also present results associated with lambda, a multivariate omnibus effect size akin to one minus a squared r. That is, lambda ranges between zero and one and smaller values indicate larger effect sizes associated with the full set of Rc's in a study. Finally, the tables also present results associated with the omnibus or pooled Rd coefficients for both the predictor and the criterion variable sets. For a given analysis and a given variable set, the pooled Rd is computed simply by adding all the Rd's for that variable set.

Isolating a Correction for Bias in Rc

The deviations in estimates of known population values of squared Rc presented in Tables 7 through 10, and the standard deviations of these estimates (akin to standard errors), were produced across 64 different research situations. These 64 different research situations involved 11 primary variations: (a) the number of variables in the predictor variable set (6, 4, 4, or 1) ("V1"); (b) the number of variables in the criterion variable set (6, 4, 2, or 2) ("V2"); (c) the total number of variables (12, 8, 6, or 12) ("VTot"); (d) the number of subjects sampled per variable (3, 10, 25, or 40) ("NCon"); (e) the total number of subjects, ranging from 18 to 480 ("NTot"); (f) the average squared bivariate correlation among variables across variable sets, i.e., the interdomain correlation coefficients ("r2terAV"); (g) the standard deviation of the interdomain bivariate correlation coefficients ("r2terSD"); (h) the average squared bivariate correlation among variables within the predictor variable sets, i.e., the intradomain correlation coefficients for the predictor variables ("r2PterAV"); (i) the standard deviation of the intradomain bivariate correlation coefficients for the predictor variables ("r2PterSD"); (j) the average squared bivariate correlation among variables within the criterion variable sets, i.e., the intradomain correlation coefficients for the criterion variables ("r2CterAV"); and (k) the standard deviation of the intradomain bivariate correlation coefficients for the criterion variables ("r2CterSD").

These 11 variations were correlated with the mean deviations in population estimates (Tables 7 through 10) derived in 1,000 samples randomly drawn in each of 64 research situations. To detect possible curvilinear relationships between these 11 variations and the mean deviations from known population parameters, squared values in each of the 11 variables were also correlated with the mean deviations from population parameters.

Thus, 22 (11 + 11) predictor variables were available. These results are reported in Table 11.

INSERT TABLE 11 ABOUT HERE.

When stepwise regression analyses were performed to select predictors of the deviations from population estimates for the parameters noted in Table 11, the number of subjects per variable ("NCon") was selected as the best predictor in all nine analyses. From among the remaining 21 predictors, the squared value of "NCon" ("NCon**2") was selected as the next best predictor in seven of the nine analyses, and as third best in two analyses. Table 12 reports regression results when "NCon" and "NCon**2" were entered first in the analyses. The table also reports the partial correlation coefficients between the remaining 20 predictor variables and the deviations in estimates from known population parameters, once the variance associated with the number of subjects per variable was removed.

INSERT TABLE 12 ABOUT HERE.

The results presented in Tables 11 and 12 clearly indicate that the best predictor of deviations from true population values of $\underline{R}c$ induced by sampling error is the ratio of the number of subjects to the number of variables. This information provides an empirical basis for isolating a correction formula for $\underline{R}c$.

Another basis for identifying a correction is theoretical. Like all parametric univariate and multivariate statistics (Knapp, 1978), the multiple correlation coefficient ($\underline{R}$) is a special case of Rc (Thompson, in press; Thompson & Borrello, 1985). Correction formulae to be applied to R have long been available to researchers (e.g., Öklin & Pratt, 1958) and are well known. For example, the researcher using the REGRESSION procedure of SPSS-X always receives an "adjusted R" calculated using a correction suggested by Wherry (1931). Carter (1979) notes that the various corrections for R tend to yield very similar results, especially when sample sizes are greater than 50. The Wherry correction can be expressed as:

1 - ((NTot-1) / (NTot-VTot-1)) * (1 - squared R)

or, equivalently, as:

squared R - ((1-squared R) * (NTot / (Ntot-VTot-1))).

The empirical findings reported in Tables 11 and 12 suggest that the ratio of the number of subjects to the number of variables in a study is the best predictor of sampling error in Rc. The Wherry correction is largely a function of exactly this ratio. Thus, both theoretical considerations and the empirical results in the present study suggest that the Wherry correction is appropriate for use with Rc, as tentatively speculated by Cliff (1987, p. 446).

The results presented in Tables 7 through 10 indicate that the sampling error in Rc gets <u>disproportionately</u> larger for each proportional decrease in the ratio of sample size to the number of variables. This conclusion is also supported by the finding that the squared ratio of subjects to variables ("NCon**2") is generally the next best predictor of bias, as indicated by the results reported Tables 11 and 12 and by the stepwise regression findings. The results also indicate that sampling error gets disproportionately larger for each proportional decrease in the size of the true population Rc. For example, as noted in Table 8, when the known population value is 0.82, the mean deviation from the true value is extremely small (a mean overestimate of 0.052) even when 12 variables are involved in the study and there are as few as three subjects per variable.

Table 13 illustrates the application of the Wherry correction to various values of squared Rc in various research situations. Table 13 illustrates that the correction is very small when Rc is very large, pretty much regardless of the ratio of subjects to variables. For a given sample size (e.g., n=18), the proportional amount of correction going from smaller to larger values of squared Rc (e.g., 0.1 to 0.3; change = 491% versus change = 127%) is smaller and smaller as Rc gets larger (e.g., 0.5 to 0.7; change = 54% versus change = 23%). When subjects are added in a given research situation, the effects of adding the first 10 subjects, for example, will be greater than the effects of adding the next 10 subjects. The results of the present study suggest that these are the desirable properties for corrections applied to Rc.

INSERT TABLE 13 ABOUT HERE.

Discussion

The results of the present study suggest three general conclusions regarding an important multivariate analytic method, a method that subsumes all other parametric significance tests as special cases (Knapp, 1978). First, the results indicate that the canonical correlation coefficient (Rc) is somewhat positively biased, i.e., sample results tend to yield overestimates of true population values.

This result is not surprising from a theoretical point of view. As Cliff (1987, p. 446) notes:

> In multiple regression we learned that R can be a substantial overestimate of P [the true population R] if the number of predictors is an appreciable fraction of the number of observations. In cancor we not only have the correlation between a variable and the [latent] composite for which the weights were optimally chosen, but the correlation between two optimally weighted composites. We would therefore expect that r* [Rc] is correspondingly likely to be even more of an overestimate...

However, the results in the present study suggest that Rc is not "correspondingly likely to be even more of an overestimate." The bias in squared Rc is minimal, unless the researcher uses a ratio of subjects to variables as small as three to one. Even with such a ludicrous subject to variable ratio, the bias is fairly minimal if true population values range as high as 0.40, as in Table 9, or 0.45, as in Table 10. Bias is almost nonexistant even with a very smal. sample size (e.g., 36, 24, 18, or 36), if the true population squared Rc is as large as 0.82, as in Table 8.

There have been a few previous Monte Carlo studies that examined the bias in Rc as at least peripheral features of the authors' purposes. For example, in a study focusing on the properties of canonical function and structure coefficients, Barcikowski and Stevens (1975) studied eight correlation matrices, drawing 100 samples for each situation under investigation. The number of variables ranged from seven to 41. The authors concluded that "for all eight examples the canonical correlations are very stable under replication... even for small sample sizes, such as 100 or 200".

In a similar vein, Dawson-Saunders (1982) conducted a Monte Carlo study focusing on correction for bias in Rd. The researcher drew 600 samples for each of 48 research situations. When the ratio of sample size to variables was five, the researcher found that the mean bias in the squared Rc for the first canonical function was 0.127 across various types of matrices.

But Dawson-Saunders (1982, pp. 141-142) concluded that, with respect to both Rc and Rd,

> bias decreases are inversely proportional to the sample size. The investigator may be less concerned with the number of variables in a study, except as this number relates to sample size, and with the degree of relationship among the variables in each individual battery.

This conclusion appears to be in error. It is exactly the ratio of number of subjects to variables that provides the best estimate of bias in estimates. The ratio is a better predictor of bias than either sample size alone ("NTot") or the interbattery correlations ("r2terAV"), as indicated by Tables 11 and 12.

These results suggest that researchers should attempt to employ at least 10 subjects per variable in multivariate studies. But even when fewer subjects are employed, results may still be interpretable if large effect sizes are detected. Researchers can be more certain of realizing such effects by solidly grounding studies in both theory and related previous empirical inquiry. Thus, the present study offers important guidance to researchers trying to determine which study features may have contributed to bias in their estimates of multivariate effects.

Second, the results reported in Tables 7 through 10 do suggest that Rd is less biased than Rc, as suggested by Cooley and Lohnes (1976, p. 212), at least when the pooled or omnibus Rd is the basis for comparison. However, this result is not surprising. As noted previously, canonical function coefficients are selected to optimize Rc, i.e., to yield the largest possible Rc for a given data set. Since maximizing Rd is not considered at all as part of the analysis, no wonder the pooled redundancy coefficient tends to be less biased than Rc!

Even though Rd tends to be somewhat less positively biased, canonical analysis does not optimize redundancy (Thompson, 1984, p. 27), and redundancy coefficients are not true multivariate statistics (Cramer & Nicewander, 1979, p. 43; Thompson, 1987). Thus, redundancy coefficients are usually not useful in canonical analysis, except in rare cases where the researcher expects to isolate "g"-functions (e.g., Sexton et al., 1988). In these rare cases, the researcher can take some comfort in knowing that neither canonical correlation coefficients (Rc) nor redundancy coefficients (Rd) tend to be very biased, especially when sample size is at least modest.

Third, the results presented in Tables 11 and 12 clearly indicate that the best predictor of bias in Rc is the ratio of the number of subjects to the number of variables. As these analyses were based on 64,000 canonical analyses of sample data, some confidence may be vested in this conclusion. Thus, the study provides an empirical basis for Cliff's (1987, p. 446) speculation that the Wherry correction, developed for use with the multiple correlation coefficient (R), may be appropriate for use with the canonical correlation coefficient (Rc).

The Wherry (1931) correction appears to be more appropriate than alternative corrections, e.g., the Olkin-Pratt correction. The Wherry correction has the advantage of being computationally simple, and yet results in corrections comparable to those provided by other formulae (Carter, 1979). The Olkin-Pratt correction will yield adjusted Rc's that are slightly higher than Rc's adjusted using the Wherry formula, and the Table 13 results do suggest that the Wherry correction is overly conservative. But the differences in the two corrections tend to be very small.

The Wherry correction appears to be too conservative, especially when sample size is small, or when Rc is small. This suggests that the correction should be used by the researcher to identify bounds within which the the true population Rc should lie. The researcher can be reasonably certain the true population values will lie between the adjusted Rc and the calculated Rc for a given canonical function. If both the adjusted and the non-adjusted canonical correlation coefficients involve effect sizes that the researcher considers noteworthy, then the researcher can be comfortable in asserting a judgment that the function is worthy of interpretation.

Even when the non-adjusted Rc is deemed noteworthy but the adjusted Rc is not, the correction has value in alerting the researcher that it may be particularly important to implement further analyses to determine which of the two estimates of Rc are most appropriate, so as to resolve the analytic ambiguity. The three relevant logics for making this determination are cross-validation (Thompson, 1984, pp. 41-47), the jackknife method (e.g., Crask & Perreault, 1977), or the bootstrap method (Diaconis & Efron, 1983).

Kerlinger (1973, p. 652) has suggested that "some research problems almost demand canonical [correlation] analysis." Similarly, Cooley and Lohnes (1971, p. 176) suggest that "it is the simplest model that can begin to do justice to this difficult problem of scientific generalization." The results in the present study suggest that researchers can determine whether their sample sizes will be adequate largely by consulting the ratio of the number of subjects to the number of variables. The results also provide empirical support for a recent speculation that Wherry's correction for the multiple correlation coefficient might be appropriately applied to the canonical correlation coefficient. Finally, the results indicate that positive bias in Rc is not as great as some researchers have supposed (e.g., Cooley & Lohnes, 1976, p. 212; Cliff, 1987, 446). Thus, canonical correlation analysis remains a potent weapon in the analytic arsenal of the behavioral scientist.

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Table 1
"Testwise" and "Experimentwise" Error Rates for Selected Studies

"Testwise" Rate	Minimum	"Experimentwise" Rate			n of Tests		Maximum
05.0%	05.0%	1 - (-	05.0%)	**	1	=	
05.0%	05.0%	1 - (	95.0%)	**	1	=	
05.0%	05.0%	1 -	95.0%			=	05.00%
05.0%	05.0%	1 - (-	05.0%)	**	5	=	22.62%
05.0%	05.0%	1 - (-	05.0%)	**	10	=	40.13%
05.0%	05.0%	1 - (-	05.0%)	**	20	=	64.15%

Note. An alpha of 0.05 equals an alpha of 05.0%. "**" means "raised to the power of". The first several rows of the table illustrate the that "testwise" and "experimentwise" error rates are the same when only one test is conducted.

Table 2
Hypothetical Observed and Latent Scores

X	Y	A	B	ZX	ZY	ZA	ZB	C1	C2	P1	P2
1	11	5	1	-1.525	1.248	-.416	.957	.303	-1.934	-.182	-.995
2	5	3	1	-1.248	-.416	-.971	.957	-.998	-.866	-.733	-1.076
3	2	2	1	-.971	-1.248	-1.248	.957	-1.578	-.212	-1.009	-1.117
4	8	8	0	-.693	.416	.416	-.957	.006	-.804	.182	.995
5	4	4	0	-.416	-.693	-.693	-.957	-.814	-.012	-.921	.832
6	12	10	1	-.139	1.525	.971	.957	1.251	-.880	1.197	-.791
7	7	6	1	.139	.139	-.139	.957	.191	.050	.094	-.954
8	1	1	0	.416	-1.525	-1.525	-.957	-1.110	1.118	-1.748	.710
9	9	12	0	.693	.693	1.525	-.957	.955	.250	1.284	1.157
10	3	7	0	.971	-.971	.139	-.957	-.346	1.319	-.094	.954
11	6	9	0	1.248	-.139	.693	-.957	.517	1.142	.457	1.035
12	10	11	1	1.525	.971	1.248	.957	1.621	.827	1.472	-.750

Note. Variables "X", "Y", "A", and "B", and their Z-score equivalents are "observed" scores. The remaining scores are "latent" or "synthetic" scores since they are created by adding together the observed scores once they have been weighted by coefficients analogous to beta weights, i.e., the canonical function coefficients.

Table 3
Canonical Results for Hypothetical Data

Variable/ Coefficient	Function I Coefficients Func	Str	Sq S	Index	Function II Coefficients Func	Str	Sq S	Index	h^2
X	.511	.499	24.87%	.470	.860	.867	75.13%	.460	1.000
Y	.867	.860	73.91%	.809	-.499	-.511	26.09%	-.271	1.000
Adequacy			49.39%				50.61%		
Redundancy			43.78%				14.24%		
Rc^2			88.63%				28.13%		
Redundancy			42.69%				14.58%		
Adequacy			48.17%				51.83%		
A	.994	.971	94.20%	.914	.147	.241	5.80%	.128	1.000
B	.242	.146	2.13%	.138	-.975	-.989	97.87%	-.525	1.000

Table 4
Bivariate Equivalents of Canonical Coefficients

Variable	Type	Variable	Type	Result
C1	Latent	P1	Latent	Function I Rc
C2	Latent	P2	Latent	Function II Rc
X	Observed	C1	Latent	Structure Coef. for X on Function I
Y	Observed	C1	Latent	Structure Coef. for Y on Function I
A	Observed	P1	Latent	Structure Coef. for A on Function I
B	Observed	P1	Latent	Structure Coef. for B on Function I
X	Observed	P1	Latent	Index Coef. for X on Function I
Y	Observed	P1	Latent	Index Coef. for Y on Function I
A	Observed	C1	Latent	Index Coef. for A on Function I
B	Observed	C1	Latent	Index Coef. for B on Function I

Table 5
Actual Population Correlation Coefficients for
Matrix #1 (Above Diagonal) and Matrix #2 (Below Diagonal)

	1	2	3	4	5	6	7	8	9	10	11	12
1		.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
2	.00		.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
3	.00	.00		.00	.00	.00	.00	.00	.00	.00	.00	.00
4	.00	.00	.00		.00	.00	.00	.00	.00	.00	.00	.00
5	.00	.00	.00	.00		.00	.00	.00	.00	.00	.00	.00
6	.00	.00	.00	.00	.00		.00	.00	.00	.00	.00	.00
7	.00	.00	.00	.00	.30	.10		.00	.00	.00	.00	.00
8	.00	.00	.00	.00	.10	.00	.00		.00	.00	.00	.00
9	.00	.00	.00	.10	.00	.00	.00	.00		.00	.00	.00
10	.00	.00	.10	.00	.00	.00	.00	.00	.00		.00	.00
11	.31	.60	.00	.00	.00	.00	.00	.00	.00	.00		.00
12	.61	.30	.00	.00	.00	.00	.00	.00	.00	.00	.00	

Note. Variable combination "A" consisted of 12 variables [set 1 = variables 1-6; set 2 = variables 7-12]; variable combination "B" consisted of 8 variables [set 1 = variables 1-4; set 2 = variables 9-12]; variable combination "C" consisted of 6 variables [set 1 = variables 1-4; set 2 = variables 11-12]; variable combination "D" consisted on 12 variables [set 1 = variables 1-10; set 2 = variables 11-12].

Table 6
Actual Population Correlation Coefficients for
Matrix #3 (Above Diagonal) and Matrix #4 (Below Diagonal)

	1	2	3	4	5	6	7	8	9	10	11	12
1		.58	.29	.10	.10	.10	.10	.10	.10	.10	.30	.60
2	.24		.30	.10	.10	.10	.10	.10	.10	.10	.30	.29
3	.24	.25		.10	.10	.10	.10	.10	.10	.10	.10	.09
4	.25	.25	.25		.10	.10	.10	.10	.10	.10	.10	.10
5	.25	.26	.26	.25		.10	.10	.10	.10	.10	.10	.10
6	.24	.25	.25	.25	.25		.10	.10	.10	.10	.10	.10
7	.25	.26	.25	.25	.25	.25		.10	.10	.10	.10	.10
8	.25	.26	.26	.26	.26	.25	.26		.10	.10	.10	.10
9	.25	.25	.25	.25	.25	.25	.25	.25		.10	.10	.10
10	.25	.26	.26	.26	.26	.26	.26	.26	.26		.30	.30
11	.25	.25	.25	.25	.25	.25	.25	.25	.25	.26		.60
12	.25	.26	.25	.25	.25	.25	.25	.25	.25	.26	.25	

Note. Variable combination "A" consisted of 12 variables [set 1 = variables 1-6; set 2 = variables 7-12]; variable combination "B" consisted of 8 variables [set 1 = variables 1-4; set 2 = variables 9-12]; variable combination "C" consisted of 6 variables [set 1 = variables 1-4; set 2 = variables 11-12]; variable combination "D" consisted on 12 variables [set 1 = variables 1-10; set 2 = variables 11-12].

Table 7
Deviations from Known Population Coefficients Across 16 Research Situations Associated with Population Matrix #1

Matrix/ NCon	I	II	III	IV	V	VI	Lambda	Redundancy Pred	Redundancy Crit
1A	.00	.00	.00	.00	.00	.00	1.00	.00	.00
3	-.447	-.289	-.173	-.085	-.030	-.004	.707	-.171	-.170
	(.091)	(.075)	(.055)	(.037)	(.021)	(.007)	(.084)	(.036)	(.036)
10	-.137	-.082	-.045	-.022	-.007	-.001	.265	-.049	-.049
	(.036)	(.023)	(.016)	(.010)	(.005)	(.001)	(.053)	(.011)	(.011)
25	-.054	-.031	-.017	-.008	-.003	.000	.109	-.019	-.019
	(.014)	(.009)	(.006)	(.004)	(.002)	(.001)	(.024)	(.004)	(.004)
40	-.033	-.019	-.010	-.005	-.002	.000	.067	-.011	-.011
	(.009)	(.005)	(.004)	(.002)	(.001)	(.000)	(.015)	(.003)	(.003)
1B	.00	.00	.00	.00			1.00	.00	.00
3	-.415	-.203	-.072	-.010			.566	-.176	-.175
	(.122)	(.084)	(.049)	(.014)			(.126)	(.055)	(.053)
10	-.125	-.056	-.019	-.003			.191	-.050	-.050
	(.043)	(.027)	(.013)	(.004)			(.058)	(.017)	(.017)
25	-.050	-.021	-.007	-.001			.078	-.020	-.020
	(.019)	(.010)	(.005)	(.001)			(.026)	(.007)	(.007)
40	-.030	-.013	-.004	-.001			.047	-.012	-.012
	(.011)	(.006)	(.003)	(.001)			(.016)	(.004)	(.004)
1C	.00	.00					1.00	.00	.00
3	-.362	-.109					.429	-.118	-.236
	(.142)	(.080)					(.149)	(.052)	(.059)
10	-.103	-.028					.128	-.033	-.066
	(.051)	(.022)					(.059)	(.016)	(.031)
25	-.041	-.011					.051	-.013	-.026
	(.021)	(.008)					(.024)	(.006)	(.012)
40	-.025	-.006					.031	-.008	-.016
	(.013)	(.005)					(.015)	(.004)	(.008)
1D	.00	.00					1.00	.00	.00
3	-.376	-.188					.491	-.056	-.281
	(.096)	(.073)					(.103)	(.016)	(.071)
10	-.114	-.053					.161	-.017	-.084
	(.037)	(.024)					(.048)	(.005)	(.026)
25	-.044	-.020					.063	-.006	-.032
	(.016)	(.009)					(.020)	(.002)	(.010)
40	-.027	-.012					.038	-.004	-.019
	(.009)	(.005)					(.012)	(.001)	(.006)

Columns I–VI are Function RcSQs.

Note. Known population parameters are reported to exactly two decimal places. Mean deviations from true population parameters across 1,000 random samples for each of the 16 research situations are reported to three decimal places and not in parentheses. Standard deviations for sample values are reported to three decimal places in parentheses.

Table 8
Deviations from Known Population Coefficients Across 16 Research Situations Associated with Population Matrix #2

Matrix/ NCon	Function RcSQs I	II	III	IV	V	VI	Lambda	Redundancy Pred	Crit
2A	.82	.09	.01	.01	.01	.01	.16	.16	.16
3	-.049	-.321	-.229	-.110	-.032	.003	.108	-.138	-.140
	(.050)	(.096)	(.076)	(.054)	(.029)	(.010)	(.024)	(.040)	(.041)
10	-.016	-.087	-.078	-.031	-.005	.007	.041	-.040	-.040
	(.029)	(.050)	(.029)	(.018)	(.010)	(.003)	(.023)	(.019)	(.018)
25	-.005	.031	-.037	-.013	.001	.008	.016	-.015	-.015
	(.019)	(.031)	(.015)	(.010)	(.006)	(.002)	(.017)	(.011)	(.011)
40	-.003	-.019	-.027	-.009	.001	.007	.010	-.009	-.009
	(.014)	(.025)	(.011)	(.007)	(.004)	(.002)	(.013)	(.008)	(.008)
2B	.82	.09	.01	.01			.16	.23	.23
3	-.045	-.245	-.111	-.008			.082	-.121	-.123
	(.061)	(.125)	(.075)	(.025)			(.042)	(.063)	(.063)
10	-.011	-.069	-.037	.004			.026	-.034	-.036
	(.036)	(.063)	(.028)	(.009)			(.031)	(.031)	(.032)
25	-.005	-.024	-.019	.005			.011	-.013	-.013
	(.023)	(.038)	(.017)	(.006)			(.021)	(.019)	(.019)
40	-.002	-.017	-.014	.005			.007	-.008	-.010
	(.018)	(.029)	(.013)	(.006)			(.016)	(.014)	(.014)
2C	.82	.09					.16	.23	.46
3	-.039	-.161					.058	-.077	-.105
	(.072)	(.151)					(.060)	(.070)	(.113)
10	-.009	-.039					.015	-.021	-.027
	(.043)	(.074)					(.040)	(.034)	(.063)
25	-.003	-.018					.005	-.008	-.011
	(.027)	(.046)					(.026)	(.022)	(.041)
40	-.003	-.012					.005	-.006	-.009
	(.021)	(.036)					(.020)	(.016)	(.032)
2D	.82	.09					.16	.09	.46
3	-.052	-.233					.077	-.048	-.148
	(.047)	(.109)					(.036)	(.023)	(.077)
10	-.014	-.067					.024	-.014	-.042
	(.029)	(.054)					(.026)	(.010)	(.044)
25	-.005	-.025					.009	-.005	-.016
	(.019)	(.032)					(.018)	(.006)	(.028)
40	-.003	-.017					.006	-.003	-.011
	(.015)	(.025)					(.014)	(.005)	(.022)

Table 9
Deviations from Known Population Coefficients Across 16 Research Situations Associated with Population Matrix #3

Matrix/ NCon	Function RcSQs I	II	III	IV	V	VI	Lambda	Redundancy Pred	Crit
3A	.40	.11	.03	.00	.00	.00	.52	.11	.11
3	-.210	-.274	-.200	-.115	-.040	-.006	.358	-.147	-.146
	(.091)	(.088)	(.071)	(.049)	(.026)	(.008)	(.061)	(.047)	(.049)
10	-.060	-.074	-.059	-.038	-.013	-.002	.137	-.044	-.043
	(.063)	(.056)	(.030)	(.018)	(.009)	(.003)	(.057)	(.024)	(.025)
25	-.020	-.026	-.022	-.018	-.006	-.001	.055	-.016	-.016
	(.042)	(.034)	(.018)	(.009)	(.004)	(.001)	(.040)	(.015)	(.014)
40	-.015	-.016	-.012	-.012	-.003	.000	.036	-.011	-.011
	(.032)	(.025)	(.014)	(.006)	(.002)	(.001)	(.031)	(.011)	(.011)
3B	.40	.07	.02	.00			.55	.13	.14
3	-.211	-.222	-.088	-.016			.303	-.144	-.144
	(.121)	(.112)	(.066)	(.021)			(.103)	(.073)	(.071)
10	-.055	-.064	-.026	-.006			.099	-.041	-.041
	(.082)	(.057)	(.028)	(.008)			(.078)	(.039)	(.039)
25	-.024	-.025	-.011	-.003			.044	-.016	-.017
	(.051)	(.034)	(.016)	(.004)			(.051)	(.023)	(.024)
40	-.014	-.017	-.007	-.002			.028	-.011	-.012
	(.040)	(.027)	(.013)	(.003)			(.039)	(.018)	(.018)
3C	.39	.06					.57	.12	.25
3	-.180	-.136					.224	-.094	-.158
	(.143)	(.127)					(.141)	(.079)	(.131)
10	-.053	-.043					.074	-.028	-.050
	(.094)	(.063)					(.095)	(.043)	(.076)
25	-.016	-.017					.025	-.009	-.016
	(.061)	(.038)					(.061)	(.026)	(.046)
40	-.011	-.013					.018	-.007	-.013
	(.047)	(.030)					(.047)	(.020)	(.036)
3D	.44	.09					.51	.07	.31
3	-.179	-.214					.242	-.051	-.185
	(.100)	(.107)					(.088)	(.032)	(.095)
10	-.049	-.066					.078	-.015	-.055
	(.064)	(.055)					(.063)	(.016)	(.057)
25	-.022	-.026					.034	-.007	-.025
	(.040)	(.034)					(.039)	(.009)	(.033)
40	-.013	-.016					.021	-.004	-.015
	(.031)	(.024)					(.031)	(.007)	(.026)

Table 10
Deviations from Known Population Coefficients Across 16 Research Situations Associated with Population Matrix #4

Matrix/			Function RcSQs					Redundancy	
NCon	I	II	III	IV	V	VI	Lambda	Pred	Crit
4A	.45	.00	.00	.00	.00	.00	.55	.17	.17
3	-.176	-.353	-.202	-.098	-.035	-.005	.379	-.133	-.132
	(.096)	(.088)	(.063)	(.042)	(.024)	(.007)	(.060)	(.060)	(.060)
10	-.050	-.108	-.057	-.026	-.009	-.001	.144	-.041	-.040
	(.065)	(.034)	(.020)	(.012)	(.006)	(.002)	(.057)	(.035)	(.033)
25	-.018	-.044	-.022	-.010	-.004	-.001	.059	-.015	-.015
	(.040)	(.014)	(.008)	(.005)	(.002)	(.001)	(.038)	(.021)	(.021)
40	-.012	-.026	-.013	-.006	-.002	.000	.037	-.010	-.010
	(.032)	(.008)	(.005)	(.003)	(.001)	(.000)	(.031)	(.017)	(.016)
4B	.33	.00	.00	.00			.67	.14	.15
3	-.223	-.254	-.090	-.011			.364	-.142	-.140
	(.122)	(.096)	(.058)	(.017)			(.107)	(.079)	(.080)
10	-.057	-.079	-.026	-.004			.121	-.041	-.040
	(.081)	(.037)	(.018)	(.006)			(.077)	(.044)	(.043)
25	-.021	-.033	-.010	-.001			.049	-.016	-.016
	(.053)	(.016)	(.006)	(.002)			(.052)	(.029)	(.028)
40	-.013	-.020	-.006	-.001			.030	-.010	-.009
	(.043)	(.010)	(.004)	(.001)			(.043)	(.023)	(.023)
4C	.23	.00					.76	.10	.15
3	-.234	-.141					.304	-.095	-.183
	(.152)	(.104)					(.155)	(.084)	(.121)
10	-.061	-.048					.094	-.028	-.053
	(.091)	(.036)					(.092)	(.046)	(.063)
25	-.025	-.019					.039	-.011	-.022
	(.062)	(.016)					(.062)	(.031)	(.042)
40	-.019	-.012					.028	-.009	-.016
	(.046)	(.009)					(.046)	(.023)	(.031)
4D	.31	.00					.69	.10	.20
3	-.216	-.240					.328	-.045	-.217
	(.105)	(.090)					(.097)	(.048)	(.084)
10	-.061	-.073					.106	-.01[illegible]	-.064
	(.068)	(.033)					(.067)	(.028)	(.050)
25	-.023	-.029					.042	-.005	-.024
	(.041)	(.013)					(.041)	(.016)	(.029)
40	-.015	-.017					.027	-.004	-.016
	(.033)	(.008)					(.033)	(.013)	(.023)

Table 11
Variations in 64 Research Situations Associated (Pearson *r*) with Deviations from True Population Values

Research Variations	Deviations from True Values								
	Rc^2 1	Rc^2 2	Rc^2 3	Rc^2 SE1	Rc^2 SE2	Rc^2 SE3	Lambda	RdPR	RdCR
NCon	.615	.746	.699	-.720	-.772	-.785	-.635	.707	.755
NTot	.556	.653	.628	-.708	-.723	-.739	-.560	.648	.680
NCon**2#	.515	.624	.586	-.631	-.662	-.662	-.537	.591	.631
NTot**2#	.425	.502	.493	-.574	-.569	-.583	-.433	.497	.521
r2terSD	.357	.031	-.059	-.126	.384	.240	-.384	.089	.145
r2terSD2#	.339	.054	-.018	-.141	.374	.211	-.371	.099	.144
r2terAV	.328	.017	.019	.172	.292	.089	-.318	.123	.137
r2terAV2#	.253	.062	.061	.108	.273	.027	-.260	.110	.119
r2CtraAV	.052	.048	.081	.288	.132	.026	-.074	.097	-.007
r2CtraAV2#	.048	.064	.115	.209	.140	.030	-.083	.107	-.013
r2PtraSD	.040	.037	.044	.267	.172	.124	-.045	-.025	.035
r2PtraAV	.038	-.013	.078	.441	.048	.026	-.006	-.010	.029
r2PtraAV2#	.035	.011	.111	.413	.075	.030	-.019	-.018	.031
r2PtraSD2#	.033	.047	.069	.270	.168	.115	-.043	-.039	.038
V2	-.029	-.210	-.330	-.126	-.071	.039	.154	-.263	.108
V2**2#	-.027	-.211	-.330	-.135	-.076	.039	.153	-.246	.100
VTot	-.011	-.189	-.330	-.251	-.121	.039	.092	.044	-.040
VTot**2#	-.010	-.183	-.330	-.248	-.119	.039	.088	.055	-.044
V1**2#	.009	-.039	-.330	-.165	-.071	.039	-.021	.241	-.122
r2CtraSD	.008	-.028	.045	.129	.083	.124	.020	-.108	.048
V1	.007	-.059	-.330	-.181	-.080	.039	-.007	.225	-.115
r2CtraSD2#	.007	-.019	.070	.133	.087	.115	.013	-.100	.044

- <u>Note</u>. Variables designated with "#" are squared values of the 11 predictor variables, used to detect curvilinear relationships of predictors with the dependent variables named in the column headings.

Table 12
Predicting Deviations from True Population Values

B- Weights	Rc^2 1	Rc^2 2	Rc^2 3	Rc^2 SE1	Rc^2 SE2	Rc^2 SE3	Lambda	RdPR	RdCR
NCon	.0166	.0174	.0111	-.0053	-.0064	-.0046	-.0233	.0086	.0130
NCon**2	.0001	-.0003	-.0002	0001	.0001	.0001	.0004	-.0001	-.0002
CONSTANT	-.2389	-.2507	-.1626	.1110	.1113	.0713	.3519	-.1230	-.1878
beta Weights									
NCon	2.271	2.742	2.557	-2.096	-2.532	-2.805	-2.229	2.609	2.786
NCon**2	-1.698	-2.048	-1.906	1.412	1.806	2.072	1.635	-1.952	-2.084
RSQ	52.30%	76.59%	67.10%	61.82%	75.92%	83.18%	53.71%	69.05%	78.72%
Partial Bivariate *r*'s									
r2terSD	.5174	.0637	-.1026	-.2046	.7817	.5841	-.5648	.1602	.3150
r2terSD2	.4905	.1126	-.0317	-.2286	.7613	.5153	-.5456	.1782	.3119
r2terAV	.4746	.0359	.0339	.2779	.5946	.2175	-.4672	.2204	.2979
r2terAV2	.3666	.1274	.1070	.1756	.5565	.0666	-.3818	.1974	.2585
r2CtraAV	.0754	.0993	.1406	.4657	.2699	.0630	-.1087	.1751	-.0144
r2CtraAV2	.0690	.1324	.2003	.3384	.2858	.0731	-.1222	.1928	-.0288
r2PtraSD	.0577	.0765	.0773	.4329	.3497	.3029	-.0659	-.0455	.0767
r2PtraAV	.0557	-.0274	.1362	.7138	.0980	.0641	-.0090	-.0175	.0625
r2PtraAV2	.0501	.0233	.1940	.6688	.1534	.0744	-.0281	-.0316	.0665
r2PtraSD2	.0479	.0968	.1208	.4370	.3427	.2816	-.0632	-.0698	.0821
V2	-.0415	-.4339	-.5754	-.2040	-.1446	.0952	.2260	-.4731	.2350
V2**2	-.0387	-.4354	-.5754	-.2186	-.1543	.0952	.2253	-.4417	.2177
VTot	-.0166	-.3915	-.5754	-.4059	-.2457	.0952	.1347	.0792	-.0857
VTot**2	-.0144	-.3785	-.5754	-.4016	-.2434	.0952	.1290	.0980	-.0947
V1**2	.0131	-.0809	-.5754	-.2669	-.1451	.0952	-.0303	.4332	-.2635
r2CtraSD	.0115	-.0586	.0791	.2082	.1694	.3026	.0300	-.1932	.1034
V1	.0104	-.1214	-.5754	-.2925	-.1627	.0952	-.0101	.4043	-.2500
r2CtraSD2	.0104	-.0401	.1221	.2154	.1769	.2808	.0189	-.1800	.0963
NTot	-.0079	-.1179	-.1872	-.2097	-.1094	.0334	.0563	.0269	-.0248
NTot**2	-.0042	-.0561	-.1005	-.1469	-.0747	.0169	.0323	.0199	-.0115

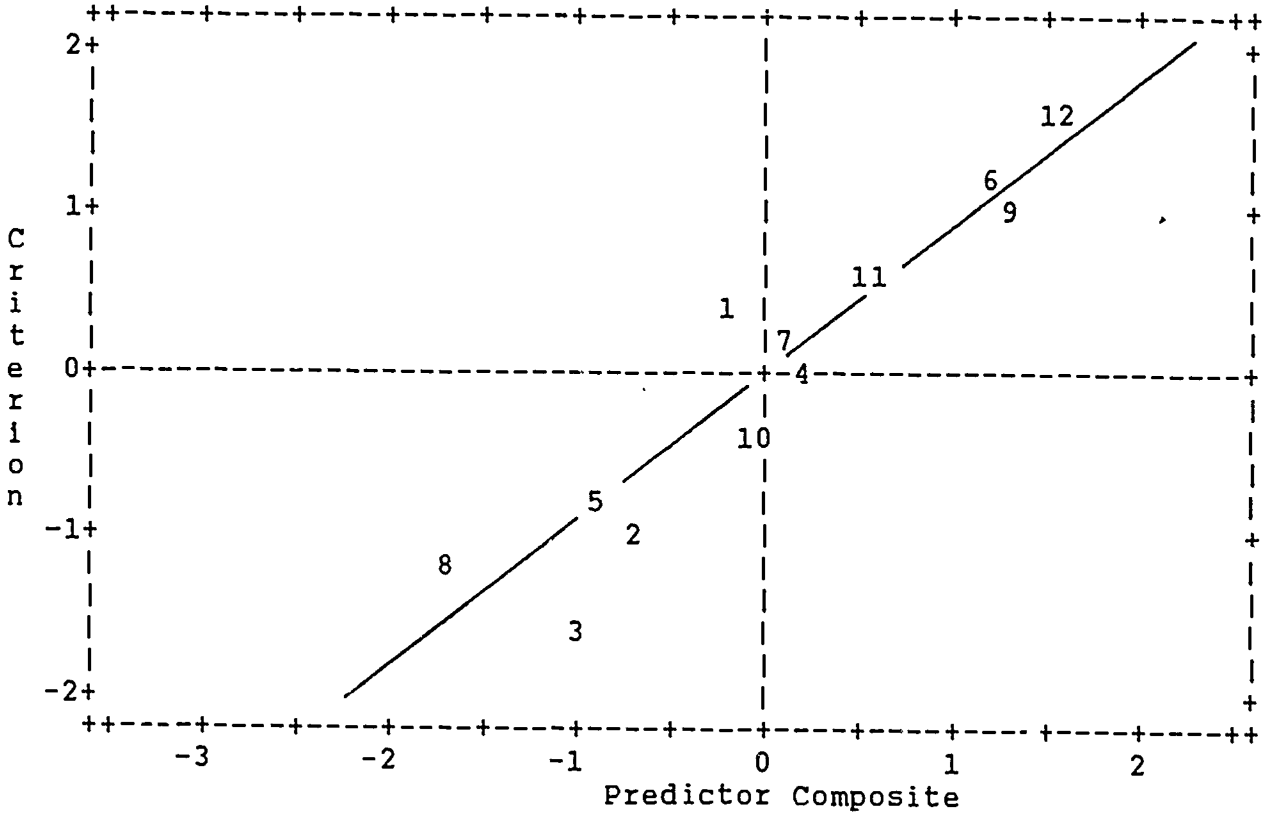
Note. The partial correlation coefficients are the correlations coefficients between the predictors and the dependent variables (deviations from true population values for the first three squared Rc's, the standard deviations of the deviations, and deviations from true values for lambda, and for Rd for both variable sets), after variance in the dependent variables associated with number of subjects per variable ("NCon") and the squared value for NCon ("NCon**2") was residualized.

Table 13
Wherry Corrections of Rc Under Various Conditions

Rcsq	v	NCon	n	Rsq*	Change	%Change
.9	12	10	120	.889	.0112	1.25%
.9	8	10	80	.889	.0112	1.25%
.9	6	10	60	.889	.0113	1.26%
.9	12	3	36	.848	.0522	5.80%
.9	8	3	24	.847	.0533	5.93%
.9	6	3	18	.845	.0545	6.06%
.7	12	10	120	.666	.0336	4.81%
.7	8	10	80	.666	.0338	4.83%
.7	6	10	60	.666	.0340	4.85%
.7	12	3	36	.543	.1565	22.36%
.7	8	3	24	.540	.1600	22.86%
.7	6	3	18	.536	.1636	23.38%
.5	12	10	120	.444	.0561	11.21%
.5	8	10	80	.444	.0563	11.27%
.5	6	10	60	.443	.0566	11.32%
.5	12	3	36	.239	.2609	52.17%
.5	8	3	24	.233	.2667	53.33%
.5	6	3	18	.227	.2727	54.55%
.3	12	10	120	.221	.0785	26.17%
.3	8	10	80	.221	.0789	26.29%
.3	6	10	60	.221	.0792	26.42%
.3	12	3	36	-.065	.3652	121.74%
.3	8	3	24	-.073	.3733	124.44%
.3	6	3	18	-.082	.3818	127.27%
.1	12	10	120	-.001	.1009	100.93%
.1	8	10	80	-.001	.1014	101.41%
.1	6	10	60	-.002	.1019	101.89%
.1	12	3	36	-.370	.4696	469.57%
.1	8	3	24	-.380	.4800	480.00%
.1	6	3	18	-.391	.4909	490.91%

Note. "Rsq*" is the canonical correlation adjusted using the Wherry correction. Negative values are treated as zero.

APPENDIX A:
Scattergram of Canonical Composite Scores on Function I



Note. The numbers plotted within the scattergram are the subject ID codes also constituting variable "X" in Table 2. For example, on Function I subject #1 had a canonical criterion composite score of .303 and a canonical predictor composite score of -.182. The slope of the bivariate regression line is Rc (0.9414).

APPENDIX B:
Raw Predictor Variable Data Which,
Together with Criterion Variable Data Presented in Tables 7-10,
Yield the Results in Tables 11 and 12

MAT	NCon	V1	V2	VTot	NTot	r2terAV	r2terSD	r2PtraAV	r2PtraSD	r2CtraAV	r2CtraSD
1A	3	6	6	12	36	.000	.000	.000	.000	.000	.000
1A	10	6	6	12	120	.000	.000	.000	.000	.000	.000
1A	25	6	6	12	300	.000	.000	.000	.000	.000	.000
1A	40	6	6	12	480	.000	.000	.000	.000	.000	.000
1B	3	4	4	8	24	.000	.000	.000	.000	.000	.000
1B	10	4	4	8	80	.000	.000	.000	.000	.000	.000
1B	25	4	4	8	200	.000	.000	.000	.000	.000	.000
1B	40	4	4	8	320	.000	.000	.000	.000	.000	.000
1C	3	4	2	6	18	.000	.000	.000	.000	.000	.000
1C	10	4	2	6	60	.000	.000	.000	.000	.000	.000
1C	25	4	2	6	150	.000	.000	.000	.000	.000	.000
1C	40	4	2	6	240	.000	.000	.000	.000	.000	.000
1D	3	10	2	12	36	.000	.000	.000	.000	.000	.000
1D	10	10	2	12	120	.000	.000	.000	.000	.000	.000
1D	25	10	2	12	300	.000	.000	.000	.000	.000	.000
1D	40	10	2	12	480	.000	.000	.000	.000	.000	.000
2A	3	6	6	12	36	.027	.085	.000	.000	.000	.000
2A	10	6	6	12	120	.027	.085	.000	.000	.000	.000
2A	25	6	6	12	300	.027	.085	.000	.000	.000	.000
2A	40	6	6	12	480	.027	.085	.000	.000	.000	.000
2B	3	4	4	8	24	.059	.120	.000	.000	.000	.000
2B	10	4	4	8	80	.059	.120	.000	.000	.000	.000
2B	25	4	4	8	200	.059	.120	.000	.000	.000	.000
2B	40	4	4	8	320	.059	.120	.000	.000	.000	.000
2C	3	4	2	6	18	.115	.150	.000	.000	.000	.000
2C	10	4	2	6	60	.115	.150	.000	.000	.000	.000
2C	25	4	2	6	150	.115	.150	.000	.000	.000	.000
2C	40	4	2	6	240	.115	.150	.000	.000	.000	.000
2D	3	10	2	12	36	.046	.110	.001	.003	.000	.000
2D	10	10	2	12	120	.046	.110	.001	.003	.000	.000
2D	25	10	2	12	300	.046	.110	.001	.003	.000	.000
2D	40	10	2	12	480	.046	.110	.001	.003	.000	.000
3A	3	6	6	12	36	.026	.060	.042	.083	.044	.089
3A	10	6	6	12	120	.026	.060	.042	.083	.044	.089
3A	25	6	6	12	300	.026	.060	.042	.083	.044	.089
3A	40	6	6	12	480	.026	.060	.042	.083	.044	.089
3B	3	4	4	8	24	.046	.087	.090	.115	.095	.124
3B	10	4	4	8	80	.046	.087	.090	.115	.095	.124
3B	25	4	4	8	200	.046	.087	.090	.115	.095	.124
3B	40	4	4	8	320	.046	.087	.090	.115	.095	.124
3C	3	4	2	6	18	.083	.111	.090	.115	.360	.000
3C	10	4	2	6	60	.083	.111	.090	.115	.360	.000
3C	25	4	2	6	150	.083	.111	.090	.115	.360	.000
3C	40	4	2	6	240	.083	.111	.090	.115	.360	.000
3D	3	10	2	12	36	.047	.079	.021	.050	.360	.000
3D	10	10	2	12	120	.047	.079	.021	.050	.360	.000
3D	25	10	2	12	300	.047	.079	.021	.050	.360	.000
3D	40	10	2	12	480	.047	.079	.021	.050	.360	.000

MAT	NCon	V1	V2	VTot	NTot	r2terAV	r2terSD	r2PtraAV	r2PtraSD	r2CtraAV	r2CtraSD
4A	3	6	6	12	36	.064	.002	.062	.003	.065	.003
4A	10	6	6	12	120	.064	.002	.062	.003	.065	.003
4A	25	6	6	12	300	.064	.002	.062	.003	.065	.003
4A	40	6	6	12	480	.064	.002	.062	.003	.065	.003
4B	3	4	4	8	24	.064	.002	.061	.002	.065	.003
4B	10	4	4	8	80	.064	.002	.061	.002	.065	.003
4B	25	4	4	8	200	.064	.002	.061	.002	.065	.003
4B	40	4	4	8	320	.064	.002	.061	.002	.065	.003
4C	3	4	2	6	18	.063	.002	.061	.002	.063	.000
4C	10	4	2	6	60	.063	.002	.061	.002	.063	.000
4C	25	4	2	6	150	.063	.002	.061	.002	.063	.000
4C	40	4	2	6	240	.063	.002	.061	.002	.063	.000
4D	3	10	2	12	36	.063	.003	.064	.003	.063	.000
4D	10	10	2	12	120	.063	.003	.064	.003	.063	.000
4D	25	10	2	12	300	.063	.003	.064	.003	.063	.000
4D	40	10	2	12	480	.063	.003	.064	.003	.063	.000

APPENDIX C:
Plot of Squared Rc for Function I with "NCon"

```
              ++----+----+----+----+----+----+----+----+----+----++
           .1+                                                   +
             |                                                   |
             |                                                   |
             |                                                   |
             |                                                   |
            0+          1              4              4          +
             |          3              8              B          |
    D        |   3      2              3              1          |
    E        |   1      6              1                         |
    V        |                                                   |
    I     -.1+          1                   Note. "B" = "11"     +
    A        |          2                                        |
    T        |          1                                        |
    I        |                                                   |
    O        |   3                                               |
    N     -.2+   1                                               +
             |   3                                               |
    F        |   1                                               |
    R        |                                                   |
    O        |                                                   |
    M     -.3+                                                   +
             |                                                   |
    P        |                                                   |
    O        |   1                                               |
    P        |   1                                               |
    U     -.4+                                                   +
    L        |   1                                               |
    A        |   1                                               |
    T        |                                                   |
    I        |                                                   |
    O     -.5+                                                   +
    N        |                                                   |
             |                                                   |
             |                                                   |
             |                                                   |
          -.6+                                                   +
              ++----+----+----+----+----+----+----+----+----+----++
              0        10        20        30        40        50
                                 n PER VARIABLE
```

This plot illustrates that deviations tend to be relatively small when the number of subjects per variable ("NCon") is at least 10, and that some deviations are small even when "NCon" is as small as three.